



SÉANCE XXXIV: VALEURS SPÉCIALES DES FONCTIONS L –À LA MÉMOIRE DE JOHN COATES

Laboratoire Analyse Géométrie Applications

Université de Paris 8 & Université Sorbonne Paris Nord

salle 100, Centre de colloques du Campus Condorcet

Mardi 28 Novembre

14:00–15:00 : *Holly Green (University of Bristol)*

On the parity conjecture for elliptic curves

I will present a new method to compute the parity of the rank of an elliptic curve and will comment on how this construction generalises to Jacobians of curves. This method involves studying the local arithmetic attached to covers of the curve. In addition, I will discuss applications to the Birch and Swinnerton-Dyer conjecture, including a new proof of the parity conjecture for elliptic curves. This is joint work with Vladimir Dokchitser, Alexandros Konstantinou, Céline Maistret and Adam Morgan.

15:00–15:30 COFFEE !

15:30–16:30 : *Mladen Dimitrov (Université de Lille)*

Uniform irreducibility of Galois action on the ℓ -primary part of Abelian 3-folds of Picard type

Half a century ago Manin proved a uniform version of Serre’s celebrated result on the openness of the Galois image in the automorphisms of the ℓ -adic Tate module of any non-CM elliptic curve over a given number field. In a collaboration with D. Ramakrishnan we provide first evidence in higher dimension. Namely, we establish a uniform irreducibility of Galois acting on the ℓ -primary part of principally polarized Abelian 3-folds with multiplication by an imaginary quadratic field having no CM factors, under a technical condition which is void in the semi-stable case.

Mercredi 29 Novembre

9:40–10:15 WELCOME! THE COFFEE IS SERVED!

10:15–11:15 : *Olivier Fouquet (Université de Besançon)*

The Iwasawa Main Conjectures with coefficient in Hecke algebras for modular motives.

The generalized Iwasawa Main Conjectures of K. Kato predict the variation of special values of L -functions in p -adic families of modular motives. I will outline a proof of them for modular motives under favorable hypotheses assuming they hold for the cyclotomic deformation and explain as a consequence the proof of some new equivariant special values formulae for the L -function of an eigencuspform.

11:30–12:30 : *Luis Garcia (UCL)*

Elliptic units for complex cubic fields

The elliptic Gamma function –an elliptic version of the ordinary Gamma function– is a meromorphic special function in three variables that mathematical physicists have shown to satisfy modular

functional equations under $SL(3, Z)$. In this talk I will present evidence (numerical and theoretical) that products of values of this function are often algebraic numbers that satisfy explicit reciprocity laws and are related to derivatives of Hecke L -functions of cubic fields at $s = 0$. We will discuss the relation to Stark's conjectures and will see that this function conjecturally allows to extend the theory of complex multiplication to complex cubic fields as envisioned by Hilbert's 12th problem. The talk will be based on arxiv:2311.04110 and is joint work with Nicolas Bergeron and Pierre Charollois.

12:30–14:30 DÉJÉNEUR

14:30–15:30 : Matteo Tamiozzo (USPN)

Some remarks on the enigmatic Tate-Shafarevich group

I will start by explaining work of Coates-Liang-Sujatha using the Iwasawa main conjecture for elliptic curves with complex multiplication to bound the corank of the p -primary part of their Tate-Shafarevich group. Then I will discuss how the techniques introduced by Bertolini-Darmon in their work on the main conjecture for elliptic curves without complex multiplication can be used to prove the visibility of p -torsion Tate-Shafarevich classes inside modular abelian varieties.

16:00–17:00 : Michael Harris (Columbia University and Institut de Mathématique de Jussieu)

Coherent cohomology and critical values of L-functions

I will report on work in progress with T. Kobayashi and B. Speh on the classification of pairs of representations of $U(V)$, $U(V')$, with V and V' hermitian spaces over the complex numbers, V' of codimension 1 in V , that admit $U(V')$ invariant linear forms that can be interpreted as cup products in coherent cohomology. By comparing this classification with the global Gan-Gross-Prasad identity (conjecture of Ichino-Ikeda and Neal Harris) one obtains relations between central values of Rankin-Selberg L -functions and automorphic periods. This in turn provides the starting point for the construction of square root p -adic L -functions in some generality.

Le séminaire de Théorie de Nombres Paris–Londres est organisé par Kevin Buzzard, Fred Diamond, Vladimir Dokchitser, Steve Lester, Yiannis Petridis, Alexei Skorobogatov (Londres) et Marc Hindry, Stefano Morra, Matthew Morrow (Paris). Le séminaire est soutenu par l'Institut de Mathématiques de Jussieu–Paris Rive Gauche, le département de Mathématiques d'Orsay, le Laboratoire Analyse Géométrie Applications, le CNRS-Imperial "Abraham de Moivre" International Research Laboratory, Cecilia Tanner Research Funding Schemes, l'Heilbronn Institute for Mathematical Research